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Modeling Long-Term Economic Development: A Python Approach to the Solow Model and Steady-State Analysis

Abstract

After economic recessions, the global economy typically undergoes a gradual recovery, with most macroeconomic factors trending upward. Robert Solow, a prominent economist of the late 20th century, introduced a growth model—later known as the Solow Model—to predict long-term macroeconomic developments based on changes in capital accumulation, labor productivity, savings rates, and other economic factors. In this paper, we utilize Python programming skills acquired throughout the semester to simulate the Solow Model and analyze how varying macroeconomic parameters influence the steady-state levels of "real output per effective labor" and "capital per effective labor" across countries.

Motivation

Economic growth is a cornerstone of national development, shaping societies and driving progress. Technological innovations have profoundly transformed not only people's lives but also the global economic landscape. This project seeks to explore how a nation's economic growth is intertwined with its technological and social advancements. By leveraging Python, we aim to model and analyze these relationships, providing insights into the dynamics of economic development.

Introduction

There are several assumptions suggested by the Solow Model. First, the economy is closed with no international trade taking place. Second, it also assumes that the accumulation of capital stocks depends solely on depreciation and yearly investment, in which the yearly investment depends solely on the saving rate. Real output (Y) is determined by a function of total factor productivity (A), net capital stocks (K), labor force (N). To explain, it can be expressed as $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$, in which α represents the capital share of income and (1 - α) represents the labor share of income. This enable us to derive the equations $K_{t+1} = K_t + I_t - dK_t$ and $I_t = s Y_t$ in which d and s respectively represents yearly fixed depreciation rate and saving rate of the economy. Lastly, in a simplified means, the model assumes a country's total population, number of workforce, and real wage all grow at a constant rate.

Given the condition stated above, the Solow Model suggested that the economy will reach a steady state of real output per effective labor ($\widehat{y}$) and capital per effective labor ($\widehat{k}$) in the long run, where the formula can be specified as $\widehat{k} = (\frac{s}{n+g+d})^{\frac{1}{1-\alpha}}$ and $\widehat{y} =(\frac{s}{n+g+d})^{\frac{\alpha}{1-\alpha}}$.

Methodology:

To address the proposed issue, we analyze different economic systems by constructing a program based on the Solow Model. In this project, we focus on the economic performance of the United States, Japan, South Korea, the United Kingdom, Canada, Germany, Mexico, and Australia, believing that these countries provide a sufficiently diverse sample. We limit our analysis to data from 2010 to 2019 to exclude the economic distortions caused by the financial crisis and the pandemic. These constraints allow for a more stable and predictable growth analysis under normal conditions.

Our goal is to develop a program that reads data for the independent variables from a .csv file and computes the dependent variables using a series of functions. Once all relevant macroeconomic variables, both independent and dependent, are gathered, we will apply them to the Solow Model equations to calculate the steady-state growth rates.

Below are the variables and formulas we use,

1)

Variable type	Total Population (N)	Real GDP (Y)	Net Capital Stock (K)	Real Wage (w)	Saving Rate (s)
Source	FRED	FRED	FRED	OECD	OECD
Unit	Million	USD (adjusted to 2016 U.S. price level)	USD (2017 U.S. price level)	USD (adjusted to 2016 U.S. price level)	% of GDP

Table 1. Independent variables

2) Dependent variables:
   - $\overline{n}$ (average population growth rate between 2010 and 2019): $\overline{\frac{\Delta N}{N}}$
   - α (capital share of economy): $1 - \overline{\frac{wN}{Y}}$
   - $\overline{g}$ (productivity growth rate): $\overline{\frac{\Delta Y}{Y}} - \alpha\overline{\frac{\Delta K}{K}} - (1 - \alpha)\,\overline{n}$
   - d (depreciation rate): $d = \overline{\frac{K_t + s_t{*}Y_t - K_{t+1}}{K_t}}$
3) Steady state rate:
   - $\hat{k} = \frac{K}{AN} = (\frac{s}{n+g+d})^{\frac{1}{1-\alpha}}$
   - $\hat{y} = \frac{Y}{AN} = (\frac{s}{n+g+d})^{\frac{\alpha}{1-\alpha}}$

Program:

File and modules:

1) Nation_info.csv
   a) consists data of 8 countries (8 lines)
   b) each line includes 2017 GDP deflator and 10-year data for population, real GDP, capital stocks, saving rate, yearly earning
2) Main.py
   a) one function for each dependent variables
   b) steady state functions
   c) main function (input: nation list, output: list of two values)
   d) output results as a table
3) Read_data.py
   a) turns line from .csv file into list
   b) splits lists into different categories for each country
4) User_interface.py
   a) allows user to input data set
   b) does calculations based on their data and print the results

Procedure:

1) Read data into separate callable lists. Nation_info.csv has 8 lines, each line has 51 elements separated by commas. Set up a function to open Nation_info.csv and read through each line in the file. Then, we turn each line from a string into a list of floats using .split() function. Lastly, create an empty list (named "complete_list") and append each list of floats into the new created list. Thus this list consists of 8 nation lists (each has 51 elements).
2) Label each string in the "complete_list" as its country name.
3) Set up functions to obtain individual lists for each type of data for a chosen nation.
4) To obtain dependent variables needed, we have a function for each variable. To calculate, we put the required variable lists for the country we are calculating as parameters. The functions will return a value which will then be stored as the value for that dependent variable.
5) Once all the needed variables for the final function are calculated, plug-in all variables into the two final equations. Each equation returns a single value which will then be outputted as a list of two variables. $\widehat{k}$ would be the first element while $\widehat{y}$ would be the second.
6) Then, we import tabulate from tabulate and print the results by combining the final results lists of every country into one table. For example, to print the $\widehat{k}$ of USA, we will simply call out the main function and let USA's list be the parameter. Then, we can print $\widehat{k}$ by selecting index [0] of the final list.
7) For the user interface, the user could enter the name of a country, and its population, GDP, capital stock, saving rate, yearly earning data. Units are specified and the time span must be uniform for all datasets. We will turn their inputs into lists of floats, which will later be used as parameters for our function. Then we will print the calculated results.

Coding techniques used:

1) Functions: to calculate all the dependents and final value.
2) For loop/while loop: to iterate through each list when summing up all the changes in certain values over years , which will later be divided by the number of years to find the average.
3) Read File: to read .csv file and store data into lists of floats.
4) Input: User interface.
5) Exceptions: to prevent zero division error or type error causes the code to break down.

Results and Analysis:

Nation	Capital/Effective Labor $\hat{k}$	Output/Effective Labor $\hat{y}$
USA	2.42055	0.862678
Japan	5.40495	0.882863
Korea	6.90469	0.89245
United Kingdom	5.44163	0.884186
Canada	5.73516	0.866101
Germany	4.35391	0.883653
Mexico	4.05976	0.897492
Australia	4.4416	0.887386

Table 2. $\hat{k}$ and $\hat{y}$ for countries selected

As shown in the table above, we calculate $\hat{k}$ and $\hat{y}$ for each of the eight countries. Now, the numerical results show some insights with respect to the country's economic condition. First of all, it shows that the US preserves the lowest level of $\hat{k}$ and $\hat{y}$ in the long run, which can be attributed to its low national saving rate of 17.22% and a high average population growth rate of 0.66%. In contrast, South Korea shows high levels of $\hat{k}$ and $\hat{y}$ in its steady state. This observation can be explained by its low average population growth rate of 0.49% and low depreciation rate of capital stocks, which results from its relatively low level of capital accumulation compared to other economies.

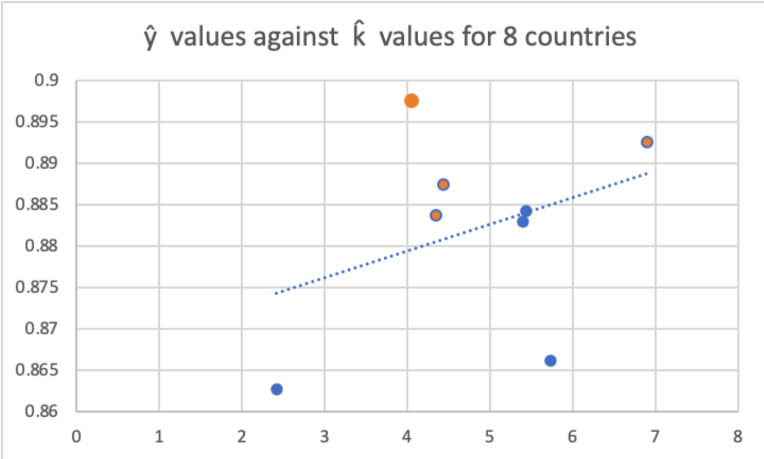

Figure 1. Regression plot of $\hat{y}$ against $\hat{k}$

The regression plot of $\hat{y}$ with respect to $\hat{k}$ shows that Germany, Mexico, Australia, and South Korea has unexpected high $\hat{y}$ based on their $\hat{k}$. Thus, the graphical result leads to an assertion that the high $\hat{y}$ values are caused by the relatively high capital income share (α) in each country, i.e. lower labor income share. Specifically to Germany and Australia, the low labor share of income in these two countries is a result of the tremendous growth of labor productivity relative to wages (Cava). According to the OECD, since 1999, Germany and Australia, just as most developed countries have experienced, underwent a lagging wage growth within its high productivity growth.

As for South Korea who experienced "the second-largest drop of the 21st countries," the prominent reason for its decline in labor share of income is marked by both sectoral shifts and "within-sector" changes (Lee et al.). The sectoral shifts were manifested by the huge increase in value-added share in its manufacturing industry with the growing importance of capital utilization. At the same time, there was a dramatic decline of the self-employment population and their income, leading to a "within-sector" plummet of labor share income.

Mexico shows the greatest discrepancy between its $\hat{k}$ and $\hat{y}$. To prevent itself from losing comparative advantage as globalization proceeds, Mexico has decreased the average wage level of domestic workers thus resulting in low labor share of income. Also, the low wage level can be attributed to the loss of labor market institutions' power ever since the 1980s in Mexico (Ibarra and Ros).

Discussion

While we run the doctest, we find out our results do not tend to lie within the 95% confidence interval of the expected steady state value. This error seems to be persistent and covert. After a few other attempts to debug and research, we figure the problem might occur at the data sets. First of all, some of the units in our data set are not consistent with the Solow Model sample we looked at. Second, we failed to convert the base year of capital stocks from 2017 to 2016. To solve the first problem, we simply have to change its unit through simple conversions. However, the second problem still exists since we are unable to access capital stocks deflator from either Fred, OECD, or any other online database. One possible solution for future improvements is to try to look up previous literary resources with related indices for us to do the conversion and reach a more accurate result. In addition, we also notice the problem that caused inaccurate results may be the equation itself. That is to say, the equations and the model are simplified under certain conditions; hence it may lead to errors especially with a short time span of data. To address this problem, one possible solution is to look up calculated indices for capital share, or alternative formulas that may output more accurate results given the data we have.

Conclusion

In this project, with the interest of determining how various macroeconomic parameters impact the steady state of an economy, we apply the concept of the Solow Model to construct a program using python to calculate the steady state of different countries. We created a program that consists of a dataset of 8 countries from 2010 to 2019. The program uses the data to run through a series of functions based on the equations derived from the Solow Model. This program then outputs the results in a table to give a clear view for comparisons and further analysis. In addition, the program also allows users to input data they have to help calculate the steady state of the country they are interested in. Our program enables users to gain better understanding and insight of how different countries perform economically in terms of the steadiness of its growth rate.

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